



Seminar

Random planar geometry with conformal invariance

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Abstract

We study random curves, loops, surfaces, fields which arise naturally in the scaling limit of a large family of statistical mechanics models in dimension two. They include Schramm-Loewner evolutions (SLE), conformal loop ensembles (CLE), Brownian loop soups, Gaussian free field (GFF), Liouville quantum gravity (LQG) and so on. These objects all possess the beautiful property of conformal invariance, and are intimately related to each other.

In the first part of our talk, we introduce the SLE, CLE and GFF. We establish various properties of these objects, and also use them to prove properties about discrete models. For example, we establish the conformal invariance for the double random current model, and compute the backbone exponent for Bernoulli percolation. The second part of our talk is focused on the Brownian loop soup. We analyze the internal structure of the clusters of the loop soup, and discuss its implications. In addition, we obtain a simultaneous coupling of the critical loop soup, CLE (4) and GFF, and extend such a coupling to the subcritical loop soup, by constructing a new family of conformally invariant fields. The last part of our talk is about random metric spaces. In particular, we study the Brownian map which is also an LQG surface with parameter $\sqrt{8/3}$. We classify the geodesic networks in the Brownian map and prove, for example, that there can be at most 9 geodesics between any two points in the Brownian map.

Date:	May 13, 2025 (Tuesday)
Time:	3:00 pm – 4:00 pm
Venue:	Room 210, Run Run Shaw Building, HKU